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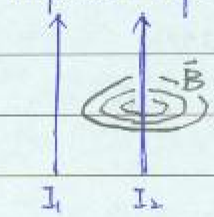
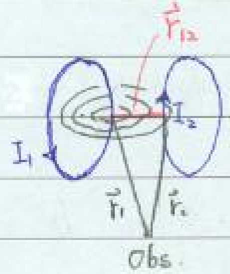
§ 5.1.3

$$F_{mag} = I \int d\vec{l} \times \vec{B}$$

Ampere's Exp.



* The law of Biot-Savart Law
From the Ampere's Law of force after performing & analyzing many experiments on the force by current carrying circuit.



* This means the force on circuit 2 due to circuit 1.

Definition of:
$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$
& The Ampere's Law can be rewritten as:

$$\vec{F}_{12} = I_2 \int d\vec{l}_1 \times \vec{B}_1 , \quad \vec{F}_{21} = I_1 \int d\vec{l}_2 \times \vec{B}_2$$

* Biot-Savart Law according experiment

$$\vec{F}_{12} = \frac{\mu_0}{4\pi} \oint \frac{(I_2 d\vec{l}_2) \times (I_1 d\vec{l}_1 \times \vec{r}_{12})}{r_{12}^3}$$

$$\equiv \alpha \cdot \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$$

$\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$ permeability of free space.

$B \equiv 1$ Tesla $= 1$ Newton / Ampere · meter

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Note : The $\vec{F}_{12}$ appears not to conform to Newton's third Law because of asymmetry of the integrated in equation.
To demostrate the relation of $\vec{F}_{12} = -\vec{F}_{21}$

* Prove : the magnetic force in releated with $\vec{F}_{12} = -\vec{F}_{21}$

Using math rule :
$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \left[\left(\frac{d\vec{l}_2 \cdot \vec{r}_{12}}{r_{12}^3}\right) d\vec{l}_1 - \frac{(d\vec{l}_1 \cdot d\vec{l}_2)}{r_{12}^3} \vec{r}_{12} \right]$$
$$\vec{B}(\vec{A}\cdot\vec{C}) = n\vec{B}$$

Calculate $\oint \frac{d\vec{l}_2 \cdot \vec{r}_{12}}{r_{12}} = \oint d\vec{l}_2 \cdot \nabla_2 \left(\frac{1}{r_{12}}\right) = 0$ P.15 P.39

then
$$\vec{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint\oint \frac{-d\vec{l}_1 \cdot d\vec{l}_2}{r_{12}^3} (\vec{r}_{12}) = -\vec{F}_{21}$$

* The final form of force :

(1) is
$$\vec{F} = \frac{\mu_0}{4\pi} \oint\oint \frac{(I_1 d\vec{l}_1)\cdot(I_2 d\vec{l}_2)}{r_{12}^3} (\vec{r}_{12})$$

(2) compare with $\vec{F} = I \oint d\vec{l} \times \vec{B}$

$$\vec{B} = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\vec{l}_1 \times \vec{r}_{12}}{r_{12}^3}, \quad d\vec{B} = \frac{\mu_0}{4\pi} I_1 \frac{d\vec{l} \times \vec{r}_{12}}{r_{12}^3}$$

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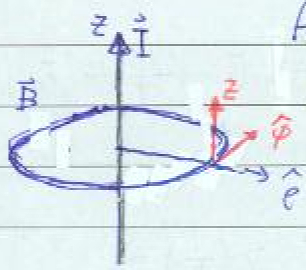
where $B_2 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\vec{l}_2 \times \vec{r}_{12}}{r_{12}^3}$ is the magnetic field produced by I_1 is known as Biot-Savart Law which was originally expressed in differential form. i.e.

$$d\vec{B}_2 = \frac{\mu_0}{4\pi} I_1 \frac{d\vec{l}_1 \times \vec{r}_{12}}{r_{12}^3}$$

§5.2 The magnetic field of steady current

$\Rightarrow$ Ampere's circuit Law (安培電路定律)



Ampere discovered the $1/\rho$ dependence of an magnetic field due to a steady current I.

$\hat{\rho} \times \vec{I}(\hat{z}) \propto -\hat{\phi}$ $\quad$ $\vec{I}(\hat{z}) \times \hat{\rho} = \hat{\phi}$

In cylindrical coordinates (ρ, φ, z) with the wire as z-axis.

Then the B-field is find as

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{I} \times \hat{\rho}}{\rho}$$

where μ_0 is a constant.
Let us find the line integral of $\vec{B} \cdot d\vec{l}$ for a path Γ_1 enclosed the wire.

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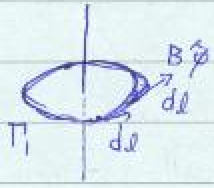
$$d\vec{l} = d\rho\,\hat{\rho} + \rho\, d\phi\,\hat{\phi} + dz\,\hat{z}$$



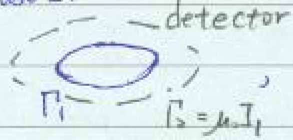
$$\frac{\vec{I}\times\hat{\rho}}{\rho} = \frac{I\hat{\phi}}{\rho_1}$$

then $\vec{B}\cdot d\vec{l} = \frac{\mu_0}{2\pi}\frac{I\hat{\phi}}{\rho}(\rho\, d\phi\,\hat{\phi}) = \frac{\mu_0 I}{2\pi\rho}\rho\, d\phi$

$$\oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\oint d\phi = \mu_0 I$$

If we want to measure Γ_1's $\oint \vec{B}\cdot d\vec{l}$
you have enclosed Γ_1

Case I.



Case II

For a path doesn't enclose the wire
$\oint_{\Gamma_2} d\phi = 0$ and so $\oint_{\Gamma_2} \vec{B}\cdot d\vec{l} = 0$

eq. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ is often called the circuital from of Ampere's Law.

Note : The origin of magnetic field
(1) Ampere's Law recognizes only one source of magnetic field : moving electric charge or steady current. $I = Q/t$

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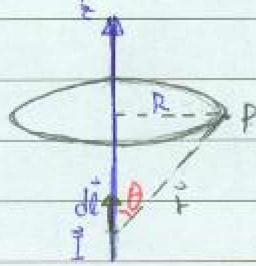
(2) The other second origin of magnetic field:

$I = \Delta Q/\Delta t$, a changing electric field

(so called a displacement current)

$dE/dt \sim dQ/dt$ Maxwell's statement.

Ex: Find the magnetic field due to a long straight current wire (infinity)



Ampere's Law: 1. The direction of I

2. " " : radius R

3. The observer of point P(r)

<1> calcute the direction cross-product

$$\frac{d\vec{l}\times\vec{r}}{r^3} \otimes = \frac{d\vec{l}\times\hat{r}}{r^2} = \frac{dl\sin\theta}{r^2}$$

Let l be the distance of the element dl from the point O, where OP is perpendicular to the wire. $R/l = \tan\theta$ or $l = R\cot\theta$

$\Rightarrow dl = -R\csc^2\theta\, d\theta$ $r^2 = R^2\csc^2\theta$

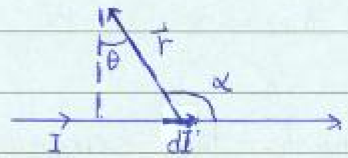
We now take θ as the independent variable

$$B = \int -\frac{\mu_0}{4\pi} I(-R\csc^2\theta)\sin\theta\, d\theta$$

$$= +\int \frac{\mu_0 I}{4\pi R}\sin\theta\, d\theta = \frac{\mu_0 I}{4\pi R}\int\sin\theta\, d\theta$$

$$B = \frac{\mu_0 I}{2\pi R}$$

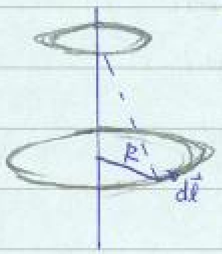
Example 5.5



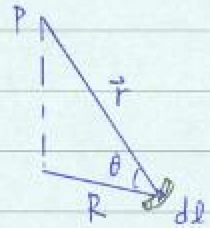
* or for finity wire

$$\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \sin\theta_2 - \sin\theta_1$$

Example 5.6



Find the magnetic field a distance z above the center of a circular loop of radius R, which carries a steady current.



$$B(z) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{dl \cos\theta}{r^2}$$

$$\cos\theta \equiv \frac{R}{(R^2+z^2)^{1/2}}$$

$$B(z) = \frac{\mu_0 I}{4\pi} \frac{R}{(R^2+z^2)^{1/2}} \int \cos\theta \, d\theta$$

* For equation of (5.39) and (5.40)

* The line integral form is

$$B(r) = \frac{\mu_0}{4\pi} \int \frac{\lambda(r) \times \hat{r}}{r^2} dr'$$

* Surface : $B(r) = \frac{\mu_0}{4\pi} \int \frac{K(r) \times \hat{r}}{r^2} d^2r'$

* Volume : $B(r) = \frac{\mu_0}{4\pi} \int \frac{J(r) \times \hat{r}}{r^2} dv'$

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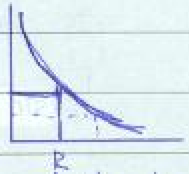
§ 5.3 The divergence of $\vec{B}$

§ 5.3.1 straight-line currents

$$B = \frac{\mu_0 I}{2\pi R}$$



It's clear that the fields has a non-zero curl for closed path (without work done)