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* $B\hat{z}$

Ex: Vector potential and magnetic field for a slowly moving charge

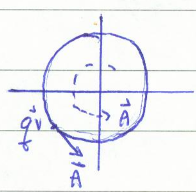
For a q moving with a constant $\vec{v}$ (steady)

$\vec{B} = \nabla \times \vec{A}$

$\vec{A}$ direction?

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}}{r} = \frac{\mu_0}{4\pi}\int \frac{\frac{dq}{dt}\,d\vec{l}}{r} = \frac{\mu_0}{4\pi}\,\frac{q\vec{v}}{r}$$

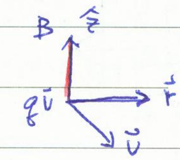
for a single charge.



Given by above equation
$\vec{B} = \nabla \times \vec{A}$ and remeber that
∇ appears differentially only on the $\frac{1}{r}$ factor.

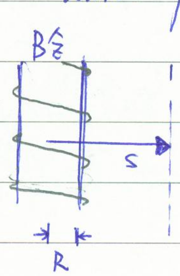
$$\vec{B} = \frac{\mu_0 q}{4\pi}\,\nabla \times \left(\frac{\vec{v}}{r}\right) = \frac{\mu_0 q}{4\pi}\,\frac{\vec{v}\times\vec{r}}{r^2}$$

direction $\hat{z}$



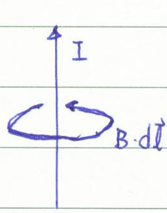
Example : 5.12

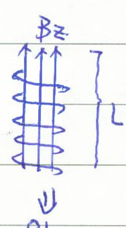
Find the vector potential of an infinite solenoid with n turns per unit length and radius R and current I.



If $s<R$ then the rule of $\vec{B} = \vec{\nabla}\times\vec{A}$ and the magnetic flux is

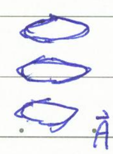
$$\int B\cdot da = \int \nabla\times\vec{A}\cdot da = \int \vec{A}\cdot d\vec{l}$$





$\int B\cdot d\vec{l} = BL$

flux $\Rightarrow$



$\Rightarrow \int \vec{A}\cdot d\vec{l}$

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Then calculate the flux area πs^2, the enclosed current $I_{en} = NI$

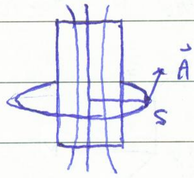
then the unit $n = N/L$, so the above term can be rewritten as

(1) $\int \vec{B}\cdot d\vec{l} = B\cdot L = \mu_0 N I \Rightarrow B = \mu_0 n I$

(2) $\int B\cdot da = \int_{loop} \vec{A}\cdot d\vec{l} \;(\hat{\phi}) \Rightarrow \mu_0 n I\cdot \pi s^2 = A\cdot 2\pi s$

$A = \frac{\mu_0 I n}{2} s\,\hat{\phi}$

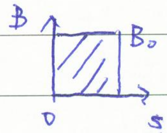
(b) if $s > R$ then enclosed total current



$\oint \vec{B}\cdot d\vec{a}$ = magnetic flux

$= \vec{B}\cdot\pi R^2 + \underbrace{\vec{B}}_{0}\cdot\pi(S-R)^2$

$= B\cdot\pi R^2$

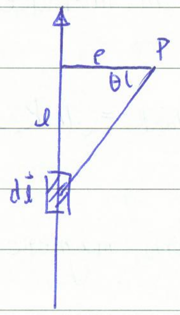


$\oint \vec{B}\cdot d\vec{a} = \oint \vec{A}\cdot d\vec{l} \Rightarrow \mu_0 I n\cdot\pi R^2 = \oint \vec{A}\cdot d\vec{l}$

$= A\cdot 2\pi s$

$\Rightarrow \vec{A} = \frac{\mu_0 I n}{2\pi s} R^2\,\hat{\phi}$

Ex : The magnetic field due to a current flowing in a long straight wire.



We now use the vector potential $\vec{A}$ to calculate $\vec{B}$ where

$$d\vec{A} = \frac{\mu_0}{4\pi} \frac{I \cdot d\vec{l}}{r}$$

$$A_z = \int_{-L}^{L} \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{\sqrt{\rho^2 + l^2}} = 2 \cdot \int_{0}^{L} \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{\sqrt{\rho^2+l^2}}$$

$$= \frac{\mu_0 I}{2\pi} \cdot \ln \left[l^2 + \sqrt{\rho^2 + l^2} \right]$$

for considering $\rho \ll L$

$$A_z = \frac{\mu_0 I}{2\pi} \ln \frac{2L}{\rho}$$

Next step for $\nabla \times \vec{A} = \begin{vmatrix} \hat{\rho} & \rho\hat{\varphi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix}$

$$B_\phi = -\frac{\partial A_z}{\partial \rho} = \frac{\mu_0 I}{2\pi\rho}$$

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§ 5.4.2 Summary. Magentic Boundary Condition (B.C)

· $\vec{B}$ - field along $d\vec{l}$ direction between an interface

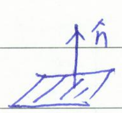
$$\oint \vec{B}\cdot d\vec{l} = (B^{\parallel}_{above} - B^{\parallel}_{below})\cdot l = \mu_0 I_{enclosed} = \mu_0 K \cdot l$$

$$\oint \vec{B}\cdot d\vec{a} = 0 \qquad B^{\perp}_{above} = B^{\perp}_{below} \quad \text{uniform magnetic flux.}$$

Then the component of B is parallel to the surface, but perpendicular to the current. Thus.

B.C. $\Rightarrow$ $B_{above} - B_{below} = \mu_0 (\vec{K} \times \hat{n})$

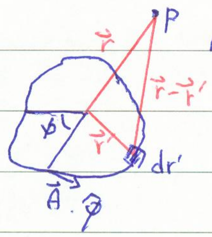
$$\vec{B} = \nabla \times \vec{A} \Rightarrow \frac{\partial A_{ab}}{\partial n} - \frac{\partial A_{be}}{\partial n} = -\mu_0 \vec{K}$$



$B^{\perp} \times \hat{n} = 0$

$B^{\parallel} \times \hat{n} \neq 0$

§5.4.3



A small current loop : The magnetic dipole $\vec{\mu}.\ \vec{m}$
$\vec{P}.\ \vec{p}$

The vector potential $\vec{A}$ may be used to advantage in determing the magnetic field $\vec{B}$. due to the small current loop.

$$\vec{A} = \frac{\mu_0}{4\pi}\oint \frac{I\, dr'}{|\vec{r}-\vec{r}'|}$$

* Note : small loop means $|\vec{r}| \gg |\vec{r}'|$
then we may expand $|\vec{r}-\vec{r}'|$ in powers of $\frac{r'}{r}$.

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|}\oint \frac{I\, dr'}{|1-\frac{\vec{r}'}{r}|}$$

Let $\frac{r'}{r} = x$, using Taylor Expansion

$$|r-r'|^{-1} = \frac{1}{r}\left[1+\frac{\vec{r}'\cdot\vec{r}}{r^2}+\cdots\right]$$ first term is zero

Then the potential is

$$\vec{A} = \frac{\mu_0 I}{4\pi}\left[\frac{1}{r^3}\oint(r'\cdot r)\,dr'\right]$$

Math rule : (1) $(\vec{r}'\times d\vec{r}')\times\vec{r} = d\vec{r}'(\vec{r}\cdot\vec{r}') - \vec{r}'(\vec{r}\cdot dr')$

(2) $d[(\vec{r}\cdot\vec{r}')\vec{r}'] = (\vec{r}\cdot dr')\vec{r}' + (\vec{r}\cdot\vec{r}')d\vec{r}'$

$d[(\vec{r}\cdot\vec{r}')\vec{r}] = (dr\cdot\vec{r}')\vec{r} + (\vec{r}\cdot\vec{r}')d\vec{r}$

(1) Replace as $(\vec{r}'\times dr')\times\vec{r} = dr'(\vec{r}\cdot\vec{r}') + (\vec{r}\cdot\vec{r}')dr'$

$(\vec{r}'\times dr')\times\vec{r} = 2(\vec{r}\cdot\vec{r}')\,dr'$

$\Rightarrow (\vec{r}\cdot\vec{r}')\,dr' = \frac{1}{2}(\vec{r}'\times dr')\times\vec{r}$

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⇒ The final form of $\vec{A}$ is

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{I}{r^3} \cdot \frac{1}{2} \oint (\vec{r}' \times d\vec{r}') \times \vec{r}$$

$$(\vec{B} = \nabla \times \vec{A}, \quad \vec{A} \Rightarrow m)$$

$$\Rightarrow \vec{A} = \frac{\mu_0 I}{8\pi} \cdot \frac{1}{r^3} \oint (\vec{r}' \times d\vec{r}') \times \vec{r}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \vec{m} \times \vec{r} \quad \text{for } \vec{m} = \frac{I}{2} \oint \vec{r}' \times d\vec{r}'$$

$\vec{m}$ as the magnetic moment of the loop which is equal to current I times the area of the loop.

* The magnetic field $\vec{B}$ is determined by calculation $\vec{\nabla} \times \vec{A}$ from equation

$$\vec{\nabla} \times \vec{A} = \nabla \times \left[\frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3} \right]$$

$$= \frac{\mu_0}{4\pi} \left[\vec{m}\, \nabla \cdot \left(\frac{\vec{r}}{r^3} \right) - (\vec{m} \cdot \nabla) \frac{\vec{r}}{r^3} \right]$$

because $\nabla \cdot \left(\frac{r}{r^3} \right) = 0$, then

$$(\vec{m} \cdot \nabla) \frac{\vec{r}}{r^3} = \left(m_x \frac{\partial}{\partial x} + m_y \frac{\partial}{\partial y} + m_z \frac{\partial}{\partial z} \right) \left(\frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{3/2}} \right)$$

$$= \frac{\vec{m}}{r^3} - \frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^5}$$

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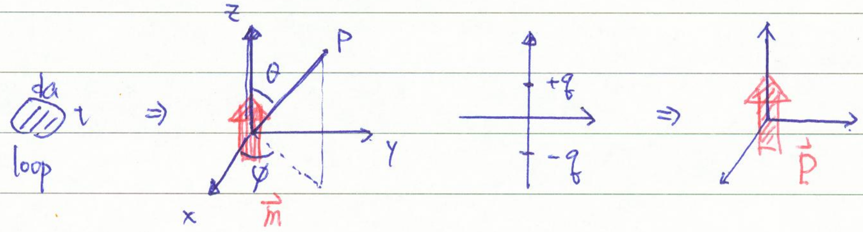
and this equation for B-field reduces to

$$\vec{B} = \nabla \times \vec{A} = -\frac{\mu_0}{4\pi}\left[\frac{m}{r^3} - \frac{3(\vec{m}\cdot\vec{r})}{r^5}\vec{r}\right] \quad \text{for small loop}$$



Home work : 5.23 , 5.26 , 5.35 , 5.36